Types of Particle Oscillations and Their Realizations in K^o and ν Oscillation

Kh. M. Beshtoev

Joint Instit. for Nucl. Reser., Joliot Curie 6, 141980 Dubna, Moscow region, Russia

Abstract

Two particle vacuum transitions (oscillations) are studied in the general case. We found that: 1) a nondiagonal mass term characterizing oscillations is the width of two particle transitions into each other (this width can be computed by the standard method); 2) two types of oscillations take place: real and virtual.

Solution of the problem of origin of mixing angle in the theory of vacuum oscillations is given.

It is shown that K^o-meson and neutrino oscillations must proceed via two stages. First, $K^o, \bar{K}^o$-eigenstates of strong interaction (or ν_e, ν_μ, ν_τ-eigenstates of weak interactions) are created. Then, owing to the strangeness violating weak interaction (or the lepton number violating interactions), these meson states (or neutrino states) are converted into superpositions of K^o_1, K^o_2-eigenstates of the weak interaction violating strangeness (or ν_1, ν_2, ν_3-eigenstates of the interaction violating lepton numbers). Further, K^o-meson or neutrino oscillations will occur in accordance with the standard scheme.

PACS: 12.15 Ff Quark and lepton masses and mixings.

PACS: 12.15 Ji Application of electroweak model to specific processes.

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1 Introduction

In the old theory of neutrino oscillations [1, 2], constructed in analogy with the theory of $K^o, \bar{K}^o$ oscillation, it is supposed that mass eigenstates are ν_1, ν_2, ν_3 neutrino states but not physical neutrino states ν_e, ν_μ, ν_τ, and that the neutrinos ν_e, ν_μ, ν_τ are created as superpositions of ν_1, ν_2, ν_3 states. This means that the ν_e, ν_μ, ν_τ neutrinos have no definite mass, i.e. their masses may vary depending on the ν_1, ν_2, ν_3 admixture in the ν_e, ν_μ, ν_τ states (naturally, in this case the low of conservation the energy and the momentum of the neutrinos is not fulfilled). Besides, every particle has mass shell and it will be left on its mass shell at passing through vacuum. Probably, this picture is incorrect one.

In this work we consider particle mixings (oscillations). At first we will consider mass matrix method for studding mixings (oscillations) two particles in general case and then come to detailed consideration of $K^o, \bar{K}^o$ and ν mixings (oscillations).

2 Probability of $a \xrightarrow{B} b$ Vacuum Transitions (Oscillations)

Let us consider two particles (states) a, b having numbers (it can be $K^o, \bar{K}^o; K^o_1, K^o_2$ or ν_e, ν_μ) which can transit each into the other. We can use the mass matrix of a, b particles for consideration of transitions between these particles in the framework of the quantum theory (or particle physics) since the mass matrix is eigenstate of a type of interaction which creates these particles (see below).

The mass matrix of a and b particles has the form

$$\begin{pmatrix} m_a & 0 \\ 0 & m_b \end{pmatrix}. \tag{1}$$

Due to the presence of a interaction violating their numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed in the following nondiagonal matrix (CP-is conserved):

$$\begin{pmatrix} m_a & m_{ab} \\ m_{ab} & m_b \end{pmatrix}, \tag{2}$$

which is diagonalized by turning through the angle β and [2]

$$tg2\beta = \frac{2m_{ab}}{\mid m_a - m_b \mid},$$

$$sin2\beta = \frac{2m_{ab}}{\sqrt{(m_a - m_b)^2 + (2m_{ab})^2}}. \tag{3}$$

$$\begin{pmatrix} m_{a'} & 0 \\ 0 & m_{b'} \end{pmatrix}.$$

$$m_{a',b'} = \frac{1}{2}\left[(m_a + m_b) \pm \left((m_a - m_b)^2 + 4m_{ab}^2\right)^{1/2}\right],$$

It is interesting to remark that expression (3) can be obtained from the Breit-Wigner distribution [3]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2} \tag{4}$$

by using the following substitutions:

$$E = m_b, \;\; E_0 = m_a, \;\; \Gamma/2 = 2m_{ab}, \tag{5}$$

where $\Gamma/2 \equiv W(...)$ is width of $a \to b$ transition, then we can use standard method [4] for computing this value.

We can see that here take place two cases of a, b transitions (oscillations): real and virtual oscillations.

1. If we consider the real transition of a into b particle then

$$sin^2 2\beta \cong \frac{4m_{ab}^2}{(m_a - m_b)^2 + 4m_{ab}^2}, \tag{6}$$

if the probability of the real transition of a particles into b particles through a interaction (i.e. m_{ab}) is very small then

$$sin^2 2\beta \cong \frac{4m_{ab}^2}{(m_a - m_b)^2} \cong 0.$$

How can we understand this real $a \to b$ transition?

If $2m_{ab} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of a particle is m_a and this mass is distributed by $sin^2 2\beta$ (or by the Breit-Wigner formula) and the probability of the $a \to b$ transition differs from zero and it is defined by masses of a and b particles and m_{ab}, which is computed in the framework of the standard method as it is pointed out above.

So, this is a solution of the problem of origin of mixing angle in the theory of vacuum oscillations.

In this case probability of $a \to b$ transition (oscillation) is described by the following expression:

$$P(a \to b, t) = sin^2 2\beta sin^2 \left[\pi t \frac{\mid m_{b'}^2 - m_{a'}^2 \mid}{2p_a}\right], \qquad (7)$$

where p_a is momentum of a particle.

2. If we consider the virtual transition of a into b particle then, since $m_a = m_b$,

$$tg 2\beta = \infty,$$

i.e. $\beta = \pi/4$, then

$$sin^2 2\beta = 1. \qquad (8)$$

In this case probability of $a \to b$ transition (oscillation) is described by the following expression:

$$P(a \to b, t) = \left[\pi t \frac{4m_{ab}^2}{2p_a}\right], \qquad (9)$$

To make these virtual oscillations real their participant in quasielastic interactions is necessary for their transitions to own mass shells [5].

It is clear that the process of $a \to b$ transition is a dynamical process and at the beginning (i.e. at t = 0) here is no superposition of a', b' particles (states).

Let us pass to consideration of concrete transitions (oscillations) between different type particles (states).

3 $K^o, \bar{K}^o$- oscillations

1) The $K^o, \bar{K}^o$-mesons, which consist of the $s, \bar{s}, d, \bar{d}$ quarks, are created in the strong interactions (the typical time of strong interactions are $t_{\rm str} \cong 10^{-23}$ s.) and are, accordingly, eigenstates of these interactions, i.e. the mass matrix of the $K^o, \bar{K}^o$ mesons is diagonal.

2) If we take into account the weak interaction (typical times of weak interactions are $t_{\rm weak} \cong 10^{-8}$ s.) which violates strangeness, then the mass matrix of K^o-mesons will become nondiagonal. If we diagonalize this matrix, then we will come to the K^o_1, K^o_2 states, which are eigenstates of the weak interaction [1].

So we can see that, if K^0-mesons are created in strong interactions, then $K^o, \bar{K}^o$ mesons are produced, and if K^o mesons are created in weak interactions then K^o_1, K^o_2 mesons are created. In second case no oscillations of K^o mesons will occur.

Now let us to give a phenomenological description of $K^o, \bar{K}^o$ meson creation and oscillation processes. We will consider the creation of $K^o, \bar{K}^o$-mesons as a quasistationary process with a typical time $t_{\rm str}$. Within of this typical time $-t_{\rm str}$, weak interactions will violate strangeness and result in the mass matrix of the K^o-mesons becoming nondiagonal. The probability for this process to occur in $t = \pi t_{\rm str}$ is:

$$W(t=\pi\Delta t_{str}) = \frac{(1-e^{-\frac{t}{\Delta t_{str}}})}{(1-e^{-\frac{t}{\Delta t_{str}}})} \simeq \pi\frac{\Delta t_{str}}{\Delta t_{weak}} \simeq \pi\cdot 10^{-15}, \qquad (10)$$

where $(1-\exp(-\frac{t}{t_{str,weak}}))$ - is the decay probability of the quasistationary state during the time $-t$.

The mass matrix of the K^o-mesons will become nondiagonal in $t = \pi 10^{-23}$ s. with a probability of $W \cong \pi 10^{-15}$. And then the K_1^o, K_2^o mesons–eigenstates of weak interactions will be created. So we can see that in this case mainly $K^o, \bar{K}^o$ mesons will be produced but not the K_1^o, K_2^o-mesons.

3) Then, when the $K^o, \bar{K}^o$ mesons, that were created in strong interactions, pass through vacuum, the mass matrix of the K^o mesons will become nondiagonal, owing to the presence of weak interactions violating strangeness. Diagonalizing it, we get K_1^o, K_2^o-meson states which are eigenstates of weak interactions. Obviously, the $K^o, \bar{K}^o$ mesons are, then, converted in to superpositions of K_1^o, K_2^o -mesons

$$K^o = \frac{K_1^o + K_2^o}{\sqrt{2}}, \qquad \bar{K}^o = \frac{K_1^o - K_2^o}{\sqrt{2}}. \qquad (11)$$

Then, oscillations of the $K^o, \bar{K}^o$ mesons will take place on a background of K_1^o, K_2^o decays. The length of these oscillations is [1, 6]:

$$L_{osc} = \frac{2.48 p_{K^o}(MeV)}{\mid m_{K_1^o} - m_{K_2^o} \mid^2 (eV)^2} \qquad (12)$$

p_{K^o} is the momentum of K^o.

The main question which arises now is: which type of oscillations real (implying actual transitions between the particles) or virtual (implying virtual transitions between particles without transition to mass shells) take place between the $K^o, \bar{K}^o$-mesons? Since the masses of K^o and $\bar{K}^o$ mesons are equal, oscillations between these mesons are real. But, if the

masses of K^o and $\bar{K}^o$ mesons were not equal, then the oscillations would be virtual (the case of K_1^o, K_2^o transitions was considered in [7]).

So, the mixings (oscillations) appear since at creating of K^o mesons are realized eigenstates of the strong interaction (i.e. $K^o, \bar{K}^o$ mesons) but not eigenstates of the weak interaction violating strangeness (i.e. K_1^o, K_2^o mesons) and then, when they pass through vacuum they are converted into superpositions of K_1^o, K_2^o mesons. If K_1^o, K_2^o mesons were originally created then mixings (oscillations) would not take place since the strong interaction conserves strangeness and isospin.

4 ν-oscillations

We can now pass to the analysis of three neutrino oscillations, taking advantage of the example of $K^o, \bar{K}^o$-meson oscillations.

1) The physical states of the ν_e, ν_μ, ν_τ neutrinos are eigenstates of the weak interaction and, naturally, the mass matrix of ν_e, ν_μ, ν_τ neutrinos is diagonal. All the available, experimental results indicate that the lepton numbers l_e, l_μ, l_τ are well conserved i.e. the standard weak interactions do not violate the lepton numbers.

2) Then, to violate the lepton numbers, it is necessary to introduce an interaction violating these numbers. It is equivalent to introducing nondiagonal mass terms in the mass matrix of ν_e, ν_μ, ν_τ. Diagonalizing this matrix we go to the ν_1, ν_2, ν_3 neutrino states. Exactly like the case of K^o mesons creating in strong interactions, when mainly $K^o, \bar{K}^o$ mesons are produced, in the considered case ν_e, ν_μ, ν_τ, but not ν_1, ν_2, ν_3, neutrino states are mainly created in the weak interactions (this is so, because the contribution of the lepton numbers violating interactions in this process is too small). And in the case 2) no oscillations take place.

3) Then, when the ν_e, ν_μ, ν_τ neutrinos pass through vacuum, they will

be converted into superpositions of the ν_1, ν_2, ν_3 owing to presence of the interactions violating the lepton number of neutrinos and will be left on their mass shells. And, then, oscillations of the ν_e, ν_μ, ν_τ neutrinos will take place according to the standard scheme [1]. Whether these oscillations are real or virtual will be determined by the masses of the physical neutrinos ν_e, ν_μ, ν_τ.

i) If the masses of the ν_e, ν_μ, ν_τ neutrinos are equal, then real oscillation of the neutrinos will take place.

ii) If the masses of the ν_e, ν_μ, ν_τ are not equal, then virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasielastic interactions, in order to undergo transition to the mass shell of the other appropriate neutrinos by analogue with $\gamma - \rho^o$ transition in the vector meson dominance model. In case ii) enhancement of neutrino oscillations will take place if the neutrinos pass through a bulk of matter [8].

So, the mixings (oscillations) appear since at neutrinos creating are realized eigenstates of the weak interaction (i.e. ν_e, ν_μ, ν_τ neutrinos) but not eigenstates of the weak interaction violating lepton numbers (i.e. (i.e. ν_1, ν_2, ν_3 neutrinos) and then, when they pass through vacuum they are converted into superpositions of ν_1, ν_2, ν_3 neutrinos. If ν_1, ν_2, ν_3 neutrinos were originally created then mixings (oscillations) would not take place since the weak interaction conserves lepton numbers.

The above considered approach for consideration of mass mixings is the mass mixings approach besides of this approach is an another approach, the charge mixings one, which is used in the vector dominance model [9].

5 Conclusion

Two particle vacuum transitions (oscillations) were studied in the general case. We found that: 1) a nondiagonal mass term characterizing oscillations is the width of two particle transitions into each other (this width can be computed by the standard method); 2) two types of oscillations take place: real and virtual.

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If K_1^o, K_2^o mesons (or ν_1, ν_2, ν_3 neutrinos) were originally created then mixings (oscillations) would not take place since the strong interaction (or the weak interaction) conserves strangeness and isospins (or lepton numbers).

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